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Technical Note

Thermal dispersion effect on MHD flow of dusty gas and dust particles through hexagonal channel

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Abstract

In the field of dynamics for a dusty fluid, the volume of the dust particles and flow behaviour of particles in different conditions is very important in engineering problems such as atmospheric fallout, nuclear reactor, powder technology, performance of solid fuel rocket nozzles, air craft icing and so many others. An analysis is presented in this paper to study the effects of thermal dispersion and Viscous dissipation on unsteady flow of a viscous incompressible dusty gas through a hexagonal channel of uniform cross section under the influence of magnetic field and time dependent pressure gradient. The results show the change in velocity profile of gas and particles in the presence of magnetic field with time, thermal dispersion and volume fraction ϕ.

1. Introduction

The laminar flow of dusty viscous fluid with different initial and boundary conditions was studied [1–4] due to its applications in Engineering Problems. Most of these researches neglected the effects of thermal dispersion and viscous dissipation on unsteady flow of viscous incompressible dusty gas with volume fraction. Rudinger [5] showed the error thus introduced from significant to large. Nayfeh [6] developed the equation of motion of dusty fluid taking into account the volume fraction of dust particles.

In the Present Paper, we study the effects of thermal dispersion and viscous dissipation on unsteady MHD flow of a dusty gas through a hexagonal channel. Explicit expressions of velocities for both the gas and dust particles have been obtained in exact form by using integral transform techniques. The effects of magnetic field and thermal dispersion on the unsteady flow of dusty gas have been shown graphically and presented in tabular form.

2. Formulation

Let us consider the effects of thermal dispersion and viscous dissipation on the motion of a dusty fluid considering volume fraction through a hexagonal channel of uniform cross section under the influence of magnetic field. Then, the governing equations are written in the following form:

$$\rho(1-\phi)\frac{\partial u}{\partial t} = (1-\phi)\left\{-\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right\} + KN_0(v-u) - \frac{\sigma\beta_0^2 u}{\rho} + \frac{1}{\rho}G_x \quad (1)$$

$$N_0 m\frac{\partial v}{\partial t} = \phi\left\{-\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right\} + KN_0(u-v) \quad (2)$$

where ρ is the density of the gas, ϕ is the volume fraction of the dust particles, N_0 is the number density of the

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particles, m is the mass of each dust particle, K is the stoke's resistance coefficient, σ is the electrical conductivity, G_x is the resultant body force on the gas. u and v represent the velocity of the gas and dust particles. Introducing the following non-dimensional quantities.

Introduce: $x = ax'$, $y = ay'$, $z = az'$, $pa^2 = \rho v p'$, $tv = a^2 t'$, $au = vu'$, $M = \frac{\sigma\beta_0^2}{\rho(1-\phi)}$ and $S = \frac{G_x}{\rho(1-\phi)}$,

$av = vv'$

Eqs. (1) and (2) become

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \varepsilon_1(v - u) - Mu + S \quad (3)$$

$$\frac{\partial v}{\partial t} = \phi'\left[-\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right] + \gamma(u - v) \quad (4)$$

where $f = \frac{mN_0}{\rho}, \phi' = \frac{\phi}{f}, \gamma = \frac{ka^2}{mv}, \varepsilon = \frac{\gamma}{(1-\phi)}, \varepsilon_1 = f\varepsilon,$ (5)

$$-\frac{\partial p}{\partial z} = f(t) \text{ (an arbitrary function of time)}$$

$S = fG_x$

and γ is the spin gradient viscosity. The corresponding non-dimensional initial and boundary conditions are

(i) $u(x, y, t) = 0 = v(x, y, t)$ for $t \leqslant 0$ (6)

(ii) $u(x, y, t) = 0$
$= v(x, y, t)$ on the boundary for $t \geqslant 0$ (7)

Let

$$x_1 = y,\ x_2 = y - \sqrt{3}x \quad \text{and} \quad x_3 = y + \sqrt{3}x \quad (8)$$

Under the transformation of Eq. (8), Eqs. (3) and (4) become

$$\frac{\partial u}{\partial t} = f(t) + \left(\frac{\partial^2}{\partial x_1^2} + \frac{4\partial^2}{\partial x_2^2} + \frac{4\partial^2}{\partial x_3^2} + 2\frac{\partial^2}{\partial x_1 \partial x_2} + 2\frac{\partial^2}{\partial x_1 \partial x_3} - 4\frac{\partial^2}{\partial x_2 \partial x_3}\right)u + \varepsilon_1(v - u) - Mu + S \quad (9)$$

$$\frac{\partial v}{\partial t} = \phi'\left(f(t) + \left[\frac{\partial^2}{\partial x_1^2} + \frac{4\partial^2}{\partial x_2^2} + \frac{4\partial^2}{\partial x_3^2} + 2\frac{\partial^2}{\partial x_1 \partial x_2} + 2\frac{\partial^2}{\partial x_1 \partial x_3} - 4\frac{\partial^2}{\partial x_2 \partial x_3}\right]u\right) + \gamma(u - v) \quad (10)$$

Subject to the initial and boundary conditions.

(i) $u(x_1, x_2, x_3, t) = 0$
$= v(x_1, x_2, x_3, t)$ for $t \leqslant 0, t \geqslant 0$ at
$x_1 = \frac{\sqrt{3}}{2},\ x_2 = \pm\sqrt{3} = x_3.$ (11)

(ii) $$\frac{\partial u}{\partial x_1} = 0 = \frac{\partial v}{\partial x_1}, \frac{\partial u}{\partial x_2} = 0 = \frac{\partial v}{\partial x_2}, \frac{\partial u}{\partial x_3} = 0 = \frac{\partial v}{\partial x_3} \quad (12)$$

We use the technique of integral transforms to solve the problem as follows:

On account of u and v being an even function of x_1 x_2 x_3 the finite Fourier sine transforms vanish. Multiplying Eqs. (9) and (10) by $\cos(P_p x_1)\cos(Q_q x_2)\cos(R_r x_3)$, and then integrate it with the limit 0 to $\frac{\sqrt{3}}{2}$, 0 to $\sqrt{3}$, and 0 to $\sqrt{3}$ and using conditions. (11) and (12), we get.

$$\frac{\partial U}{\partial t} = a_{pqr} f(t) - b_{pqr} U + \varepsilon_1(V - U) - MU + S, \quad (13)$$

$$\frac{\partial V}{\partial t} = \phi^1\left[a_{pqr} f(t) - b_{pqr} U\right] + \gamma(u - v), \quad (14)$$

where $a_{pqr} = \frac{(-1)^{p+q+r}}{P_p Q_q R_r}$ and $b_{pqr} = P_p^2 + Q_q^2 + R_r^2$ (15)

and

$$u = v = \int_0^{\sqrt{3/2}} \int_0^{\sqrt{3}} \int_0^{\sqrt{3}} u(x_1, x_2, x_3, t)\cos(P_p x_1) \cos(Q_q x_2)\cos(R_r x_3)\, dX_1\, dX_2\, dX_3 \quad (16)$$

Solving Eqs. (13) and (14) by using Laplace transforms under the transformed initial condition $U = 0 = V$ and $t = 0$, we get

$$S\overline{U} = a_{pqr}\bar{f}(S) - b_{pqr}\overline{U} = \varepsilon_1(\overline{V} - \overline{U}) - M\overline{U} + S \quad (17)$$

$$S\overline{V} = \phi'[a_{pqr}\bar{f}(S) - b_{pqr}\overline{U}] + \gamma(\overline{U} - \overline{V}) \quad (18)$$

where $\overline{U}$, $\overline{V}$, $\bar{f}(s)$ are the Laplace transform of respective quantities.

$$\overline{U} = \frac{a_{pqr} f'(S)}{(S_1 - S_2)}\left[\frac{S_1 + \varepsilon}{S - S_1} - \frac{S_2 + \varepsilon}{S - S_2}\right] \quad (19)$$

$$\overline{V} = \frac{a_{pqr}\bar{f}(S)}{(S_1 - S_2)}\left[\frac{S_1\phi' + \varepsilon}{S - S_1} - \frac{S_2\phi' + \varepsilon}{S - S_2}\right] \quad (20)$$

S_1 and S_2 are two roots of quadratic equation

$$S^2 + (b_{pqr} + \varepsilon_1 + \gamma + M - S) + \varepsilon b_{pqr} = 0. \quad (21)$$

and hence,

$$S_1 = -\frac{1}{2}\left[(b_{pqr} + \varepsilon_1 + \gamma + M - S) + \sqrt{\left\{(b_{pqr} + \varepsilon_1 + \gamma + M - S)^2 - 4\varepsilon b_{pqr}\right\}}\right] \quad (22)$$

$$S_2 = -\frac{1}{2}\left[(b_{pqr} + \varepsilon_1 + \gamma + M - S) - \sqrt{\left\{(b_{pqr} + \varepsilon_1 + \gamma + M - S)^2 - 4\varepsilon b_{pqr}\right\}}\right] \tag{23}$$

By applying convolution theorem and put $f(t) = C$, and C is an absolute constant, we get

Table 1
Variation of U/C and V/C for various values of Φ, X, M, and S

S	X	M	Φ	U/C	V/C
0	0	5	0	0.1586	0.1499
			0.04	0.1595	0.1518
			0.08	0.1604	0.1536
		10	0	0.1498	0.1393
			0.04	0.1511	0.1417
			0.08	0.1524	0.144
	0.2	5	0	0.1474	0.1394
			0.04	0.1482	0.1412
			0.08	0.1491	0.1429
		10	0	0.1394	0.1298
			0.04	0.1406	0.132
			0.08	0.1418	0.1341
1	0	5	0	0.1604	0.1522
			0.04	0.1612	0.154
			0.08	0.162	0.1557
		10	0	0.1517	0.1416
			0.04	0.153	0.1438
			0.08	0.1542	0.1461
	0.2	5	0	0.1491	0.1415
			0.04	0.1498	0.1432
			0.08	0.1505	0.1447
		10	0	0.1412	0.1319
			0.04	0.1423	0.134
			0.08	0.1434	0.136
2	0	5	0	0.1621	0.1545
			0.04	0.1629	0.1561
			0.08	0.1636	0.1577
		10	0	0.1537	0.1438
			0.04	0.1548	0.146
			0.08	0.156	0.1482
	0.2	5	0	0.1506	0.1436
			0.04	0.1513	0.1452
			0.08	0.1519	0.1466
		10	0	0.1429	0.1339
			0.04	0.144	0.136
			0.08	0.145	0.1379

$$u = \frac{16C}{3\sqrt{3}} \sum_{p=q=r=0}^{\infty} \frac{a_{pqr}}{b_{pqr}} \left[1 - \frac{1}{S_1 - S_2}\{(S_1 + b_{pqr})e^{S_2 t} - (S_2 + b_{pqr})e^{S_1 t}\}\right] \cos(P_p X_1)\cos(Q_q X_2)\cos(R_r X_3) \tag{24}$$

$$v = \frac{16C}{3\sqrt{3}} \sum_{p=q=r=0}^{\infty} \frac{a_{pqr}}{b_{pqr}} \left[1 - \frac{1}{S_1 - S_2}\{(S_1 + \phi' b_{pqr})e^{S_2 t} - (S_2 + \phi' b_{pqr})c^{S_1 t}\}\right] \cos(P_p X_1)\cos(Q_q X_2)\cos(R_r X_3) \tag{25}$$

3. Discussion

Table 1 and Figs. 1–4 are self explanatory. Here, the variations of velocity of dust and gas particles for various values of the parameters such as the thermal dispersion parameter S, the magnetic parameter M, and the volume fraction ϕ are shown. These figures and the table clearly show that the gas particles move faster than the

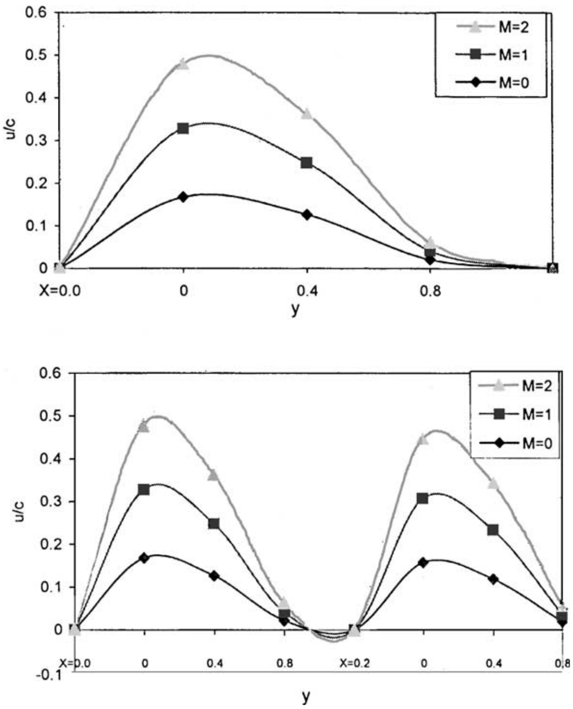

Fig. 1. Velocity profile for $f = 1.0$, $g = 5.66$, $S = 1$ at $M = 0, 5, 10$.

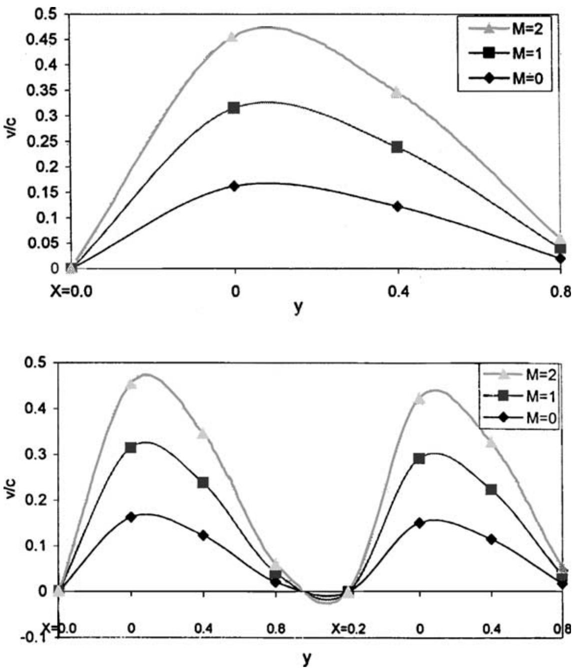

Fig. 2. Velocity profile for $f = 1.0$, $g = 5.66$, $S = 1$ at $M = 0, 5, 10$.

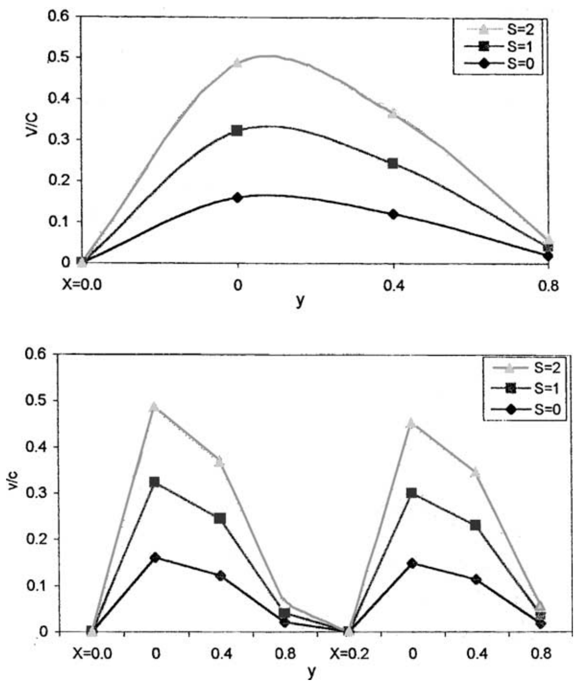

Fig. 4. Velocity profile for $f = 1.0$, $g = 5.66$, $M = 0$ at $S = 0, 1, 2$.

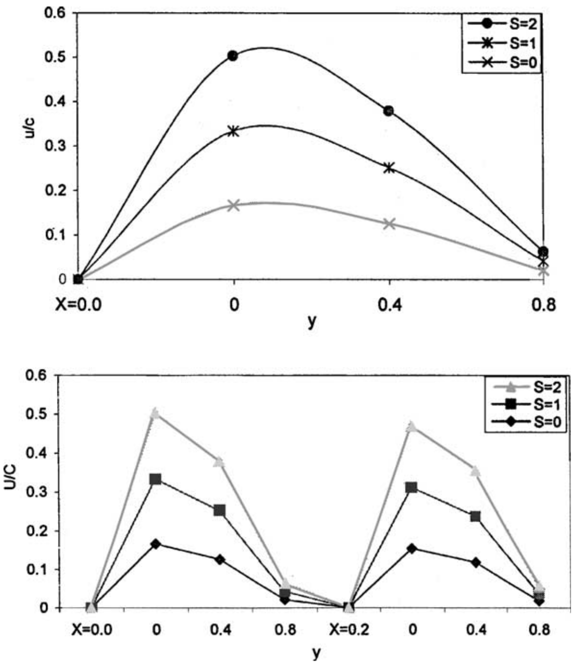

Fig. 3. Velocity profile for $f = 1.0$, $g = 5.66$, $M = 0$ at $S = 0, 1, 2$.

dust particles, whereas the increasing values of the magnetic parameter M decelerate the velocities of both the gas and the dust particles, on the other hand increasing values of thermal dispersion parameter S increase the velocities of both the gas and the dust particles. Also the similar effect on the velocities of both the gas and dust particles is observed when the volume fraction ϕ increases.

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